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Performance of OFDM Systems Using Nonlinear Companding Transform

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Abstract— Orthogonal Frequency Division Multiplexing (OFDM) is an efficient method of data transmission for high speed communication systems. The main drawback of OFDM system is the high Peak to Average Power Ratio (PAPR) of the transmitted signals, which reduces the efficiency of transmit high power amplifier. PAPR is one of the serious problems in any wireless communication system using multi carrier modulation technique like OFDM. Coding, phase rotation, clipping etc. are among many PAPR reduction schemes that have been proposed to overcome this problem. In this paper, a novel scheme is proposed for PAPR reduction. The key idea is to transform the original Gaussian-distributed OFDM signals into a specific statistics form. Significant reduction in PAPR has been achieved using this technique and this enables more flexibility in the companding form. A favorable tradeoff between PAPR reduction and BER performance can be achieved by properly choosing the transform parameters. The significance and accuracy of the analytical expressions for transform gain in PAPR, complementary cumulative density function (CCDF), and selection of transform parameters are justified by the simulation results.

Index Terms— Complementary Cumulative Distribution Function (CCDF), Fast Fourier Transform (FFT), Inverse Fast Fourier Transform (IFFT), Intersymbol interference (ISI), Multi-carrier modulation (MCM), Nonlinear Companding transform (NCT), Orthogonal frequency division multiplexing (OFDM), Peak-to-average power ratio (PAPR), Probability density function (PDF).

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1 INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is an attractive candidate and due to its high spectrum efficiency and robustness to the multipath [1], it is one of the most competitive techniques for fourth generation (4G) wireless communication systems. In OFDM the total bandwidth is divided into several narrow band sub-channels. A high rate data stream is split into a number of lower rate streams and is transmitted in parallel over the subcarriers. To minimize the intersymbol interference (ISI) effect, the symbol duration of each sub channel is made relatively larger than the delay spread. By introducing a guard time in every OFDM symbol, ISI can almost be eliminated. It also eliminates the need for equalizers and efficient hardware implementation can be realized using Fast Fourier transform (FFT) techniques. However, one major drawback is the occasionally high peak-to-average power ratio (PAPR) of the transmitted signals. High PAPR has been recognized as one of the major practical problems involving OFDM modulation. This problem results from the nature of the modulation itself, where multiple subcarriers are added together to form the signal to be transmitted. The OFDM systems are constrained to a limited peak power due to the limitation of the dynamic range over which the transmitter high power amplifier operates linearly. PAPR results in the saturation of HPA. Hence the system devices, like high power amplifiers, A/D converters, D/A converters etc. must have extremely large dynamic range to avoid the nonlinear distortion, which will severely reduce the system performance. PAPR reduction techniques are therefore of great importance for OFDM systems. Various PAPR reduction techniques have been proposed for OFDM systems [2]. Some of them are clipping and filtering [3], selective mapping (SLM) technique [4], partial transmit sequence (PTS) [5], active constellation extension (ACE) [6], and companding transform [7],[8],[9],[10],[11],[12],[13] etc.

One of the most pragmatic and easiest approaches is clipping and filtering which can snip the signal at the transmitter to eliminate the appearance of high peaks above a certain level. But due to non-linear distortion introduced by this process, orthogonality [3] is destroyed to some extent, which results in in-band and out-band noises. In-band noise cannot be removed by filtering and it decreases the bit error rate (BER). Out-band noise reduces the bandwidth efficiency but frequency domain filtering [15] can be employed to minimize the out-band noise power. Although filtering has a good effect on noise suppression, it may cause peak re-growth. To overcome this drawback, the whole process is repeated several times until a desired situation is achieved. In PTS the data is grouped into clusters and each of them is procedded with a smaller IFFT. The subcarriers in each subblock are rotated by the same phase factor such that the PAPR of the combination is minimized. SLM is based on selecting one of the transformed blocks for each data block, which has the lowest PAPR. Even though there is no distortion due to PTS and SLM techniques, side information per OFDM symbol is required and that should be received without errors. The side information has to be heavily protected. Also the amount of PAPR reduction depends on the subblock partitioning and the

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design of the phase sequences. In ACE some of the outer signal constellation points are extended towards outside of the constellation such that the PAPR of the resulting block is reduced. But this will increase the average energy per bit and might be higher than the NL distortion reduction and reduce the BER performance improvement. Also the larger the constellation size the lower the number of extensible points. Among these existing techniques, companding transform is an attractive solution due to its high efficiency and low implementation complexity.

A non-linier companding technique for PAPR reduction which employed a logarithmic-based μ-law companding was first described in [7], and showed to be rather effective than the 'hard' clippings. Later, the necessity of taking into account the statistical characteristics of the OFDM signal was first indicated by Huang, X *et.al.* [8]. Up to now, a lot of works have been done to design a desirable distribution form of the transformed signals, e.g. the exponential companding (EC) [9], piecewise companding (PC) [10], and the trapezium or trapezoidal companding (TC) in [11],[12]. The distribution of large amplitude signals is found to be very high in the EC scheme, which makes it unsuitable under certain bit error rate performance constraints. The lack of necessary flexibility in the companding forms is another issue for the EC and PC schemes. An effective tradeoff between the PAPR and BER performance is offered by the TC scheme [11], but its companding function is more complicated than others.

This paper proposes and evaluates a novel NCT scheme. In this scheme the original OFDM signals are transformed into a specific statistics form. By introducing the variable transform parameters and an inflexion point in the target probability density function (PDF), the proposed scheme can achieve an effective PAPR reduction as well as an improved overall performance. From the results it is clear that by properly choosing the parameters the impact of companding distortion can be significantly reduced. In addition, this scheme enables more flexibility and freedom in the companding form so that a favorable tradeoff between the PAPR reduction and BER performance can be achieved. Simulations results demonstrate the effectiveness and robustness of the proposed scheme.

This paper is structured as follows. Characterization of the OFDM system using the proposed NCT is discussed in Section 2. Section 3 describes the proposed new NCT scheme and Section 4 describes the corresponding theoretical analysis in terms of the transform gain in PAPR, complementary cumulative density function (CCDF) and impact of companding distortion. In Section 5, simulation results are given and compared with the existing techniques. Section 6 gives the conclusion.

2 PAPR IN OFDM SYSTEM

In OFDM system, multiple sub-carriers are superimposed to obtain the output. Hence some instantaneous power output might increase to a large extent and become far higher than the mean power of the system. If the peak power is too high, it could be out of the scope of the linear range of the power amplifier. This gives rise to non-linear distortion and resulting in performance degradation. If no measure is taken to reduce the high PAPR, OFDM systems could face serious restriction for practical applications.

Fig. 1 shows the block diagram of a baseband OFDM system using NCT technique for PAPR reduction. An OFDM symbol is made of sub-carriers modulated by constellations mapping. This mapping can be achieved from Binary Phase-Shift Keying (BPSK) or Quadrature Amplitude Modulation (QAM). For an OFDM system with N sub-carriers, the complex-value symbol vector $X = [X_0, X_1, \ldots, X_{N-1}]T$, is obtained from the input bit stream by mapping into BPSK or QAM. In the discrete-time domain, the over-sampled OFDM symbol vector is, $x = [x_0, x_1, \ldots, x_{lN-1}]T$. The nth element can be expressed as:

$$x_n = \frac{1}{\sqrt{lN}} \sum_{k=0}^{N-1} X_k . e^{\frac{j2\pi nk}{lN}} , n = 0,1,\ldots,lN-1 \qquad (1)$$

where l is over-sampling factor. OFDM symbol vector x can be achieved by performing a lN-point Inverse Fast Fourier Transform (IFFT) operation to X with $(l-1)N$ zero-padding.

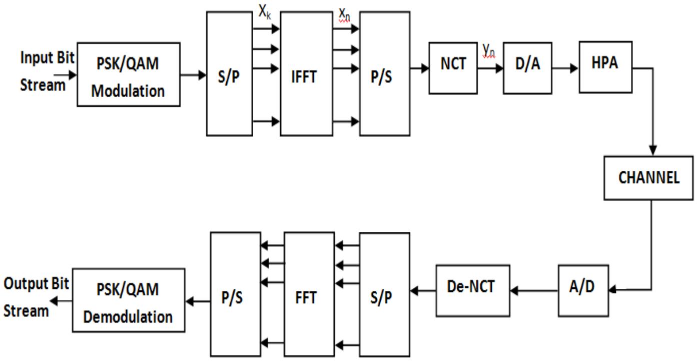

Fig. 1. Block diagram of a baseband OFDM system using NCT technique

Consider that symbols X_k are statistically independent and identically distributed (i.i.d). According to central limit theorem, when N is large, the real and imaginary parts of x_n become Gaussian distributed, each with zero mean, variance σ^2 and the signal amplitude $|x_n|$ follows Rayleigh distribution.

The PAPR of OFDM signal is defined as the ratio of the maximum instantaneous power to the average power, i.e.

$$PAPR_x = \frac{P_{PEAK}}{P_{AVERAGE}} = \frac{\max\limits_{0 \le n \le lN-1} \left\{ |x_n|^2 \right\}}{E\left\{ |x_n|^2 \right\}} \qquad (2)$$

where $E\{\cdot\}$ and $max\{\cdot\}$ denote the mathematical expectation and maximal element function, respectively.

In the proposed PAPR reduction technique using NCT, the original signal x_n is transformed according to the companding function $h(\cdot)$. This operation transforms each OFDM signal sample one at a time and which only changes the amplitude of input signal. The transformed signal $y = [y_0, y_1, \ldots, y_{lN-1}]T$ can be expressed as

$$y_n = h(x_n), n = 0, 1, \ldots, lN-1. \qquad (3)$$

At the receiver side, the inverse function $h^{-1}(\cdot)$ is used as the de- companding function.

3 PROPOSED SCHEME

Reallocation of both the power distribution and statistical characteristics of the transmitted OFDM signal with more reasonability and flexibility is the first step in the proposed scheme. This will give more effective PAPR reduction with low complexity and moderate performance degradation. In this scheme the original Gaussian-distributed signal x_n is transformed into a specific statistics form, which is defined by a piecewise function in the interval *[0, A]*, where $A > 0$. For the output, |yn|, the target PDF with the inflexion point *cA* with ($0 < c < 1$) is:

$$f_{|y_n|}(x) = \begin{cases} kx^m, 0 \le x \le cA \\ k(cA)^m, cA < x \le A \end{cases} \tag{4}$$

The parameters *k* and *m* are variable positive numbers. The ultimate companding transform with the average output power adjustment can be specified by these parameters. From the definition of PDF we have,

$$\int_{-\infty}^{\infty} f_{|y_n|}(x)dx = 1 \tag{5}$$

Then *k* can be derived as:

$$k = \frac{m+1}{c^m A^{m+1}(m+1-mc)} \tag{6}$$

To keep the power level of the transform as unchanged, $E\{|y_n|^2\} = E\{|x_n|^2\}$. By making appropriate substitutions to this condition we can obtain the expression for *A* as follows:

$$A = \left(3\sigma^2 \frac{m+3}{m+1} \cdot \frac{m(1-c)+1}{m(1-c^3)+3}\right)^{\frac{1}{2}} \tag{7}$$

The non-linear companding transform *h(.)* should be a strict monotonically increasing function. Hence the following identity can be used to calculate *h(x)*.

$$h(x) = \mathrm{sgn}(x).F_{|y_n|}^{-1}[F_{|x_n|}(x)] \tag{8}$$

where *sgn(x)* be the signum function, $F_{|y_n|}^{-1}(.)$ is the inverse CDF function of the output signal amplitude $|y_n|$ and $F_{|x_n|}(x)$ be the CDF of the input signal amplitude $|x_n|$. The cumulative distribution function (CDF) of the input signal amplitude $|x_n|$ can be derived as:

$$\begin{aligned} F_{|x_n|}(x) &= \mathrm{Prob}\{|x_n| \le x\} \\ &= \int_0^x \frac{2y}{\sigma^2} e^{-\frac{y^2}{\sigma^2}} dy \\ &= 1 - e^{-\frac{x^2}{\sigma^2}}; x \ge 0 \end{aligned} \tag{9}$$

The CDF of the output signal amplitude $|y_n|$ be:

$$F_{|y_n|}(x) = \begin{cases} \dfrac{kx^{m+1}}{m+1}, 0 \le x \le cA \\ k(cA)^m x - \dfrac{km(cA)^{m+1}}{m+1}, cA < x \le A \\ 1, x > A \end{cases} \tag{10}$$

Now the inverse $F_{|y_n|}^{-1}(.)$ can be derived as:

$$F_{|y_n|}^{-1}(x) = \begin{cases} \left(\dfrac{m+1}{k}\right)^{\frac{1}{m+1}}, x \le \dfrac{k(cA)^{m+1}}{m+1} \\ \dfrac{x}{k(cA)^m} + \dfrac{mcA}{m+1}, x > \dfrac{k(cA)^{m+1}}{m+1} \end{cases} \tag{11}$$

Now *h(x)* can be derived as:

$$h(x) = \begin{cases} \mathrm{sgn}(x)\left(\dfrac{m+1}{k}\left(1-\exp\left(-\dfrac{|x|^2}{\sigma^2}\right)\right)\right)^{\frac{1}{m+1}}, |x| \le \chi_0 \\ \mathrm{sgn}(x)\left(\dfrac{1-\exp\left(-\dfrac{|x|^2}{\sigma^2}\right)}{k(cA)^m} + \dfrac{mcA}{m+1}\right), |x| > \chi_0 \end{cases} \tag{12}$$

where $\chi_0 = \sigma\left(-\ln\left(1-\dfrac{k(cA)^{m+1}}{m+1}\right)\right)^{\frac{1}{2}}$

At the receiver section a corresponding de-companding function h^{-1}*(x) is* required, which can be described as:

$$h^{-1}(x) = \begin{cases} \mathrm{sgn}(x)\sigma\left(-\ln\left(1-\dfrac{k|x|^{m+1}}{m+1}\right)\right)^{\frac{1}{2}}, |x| \le cA \\ \mathrm{sgn}(x)\sigma\left(-\ln\left(1-k(cA)^m\left(|x|-\dfrac{mcA}{m+1}\right)\right)\right)^{\frac{1}{2}}, |x| > cA \end{cases} \tag{13}$$

With the proposed scheme the final PAPR of the transformed signal *y* can be given by:

$$PAPR_y = \frac{\max\limits_{0 \le n \le N-1}\{|y_n|^2\}}{E\{|y_n|^2\}} = \frac{1}{\sigma^2}\left(\frac{1-\exp\left(-\dfrac{A_{i\max}^2}{\sigma^2}\right)}{k(cA)^m} + \frac{mcA}{m+1}\right)^2 \tag{14}$$

where A_{imax} is the peak amplitude of input signal.

$$A_{i\max} = \max_{0 \le n \le lN-1}\{|x_n|\} \tag{15}$$

Since PAPR is a random variable, it is more helpful to describe the PAPR in terms of complementary cumulative distribution function (CCDF), since PAPR is a random variable. It can be formulated as the probability that the PAPR of the input OFDM signal *x* exceeds a threshold $PAPR_0 = \Gamma$.

$$CCDFx\ (\Gamma) = Prob\ \{PAPRx > \Gamma\} \approx 1 - (1 - exp(-\Gamma))N \tag{16}$$

The CCDF of the transformed signal is given by:

$$CCDFy\ (\Gamma) = Prob\ \{PAPRy > \Gamma\} \tag{17}$$

Substituting from above equqtion we get:

$$CCDFy\ (\Gamma) = CCDFx\left(\frac{k^2(cA)^{2m}(m+1)^2 A_{i\max}^2}{\left((m+1)\left(1-\exp\left(-\dfrac{A_{i\max}^2}{\sigma^2}\right)\right)+km(cA)^{m+1}\right)^2}\Gamma\right) \tag{18}$$

Generally the companding transform is an extra nonlinear process applied to the transmitted signal. Hence at the receiver it results in BER performance degradation. Thus, the transformed signal can be modeled as the summative of an attenuated signal component and companding noise, b_n. This modeling can be derived from the extension of the Bussgang theorem, according to which the real or complex Gaussian signals can be represented as the sum of a useful attenuated input replica and an uncorrelated nonlinear distortion noise. Now the model of the transformed signal is:

$$y_n = \alpha \,.\, x_n + b_n, \quad n = 0, 1, \dots, lN - 1 \qquad (19)$$

where α is attenuation coefficient. In [14], α has been given by:

$$\alpha = \frac{E\{y_n x_n^*\}}{E\{x_n x_n^*\}} = \frac{1}{2\sigma^2}\int_0^\infty x h(x) f_{|x_n|}(x)dx \qquad (20)$$

where $f_{|x_n|}$ is the PDF of $|x_n|$.

4 THEORETICAL ANALYSIS

Characterization of the theoretical performance of the proposed scheme is described in this section. The theoretical analysis can be performed by means of the achievable transform gain in PAPR and impact of companding distortion on the BER performance. Fig. 2 describes the transfer curves of the companding function $h(x)$ with various parameters.

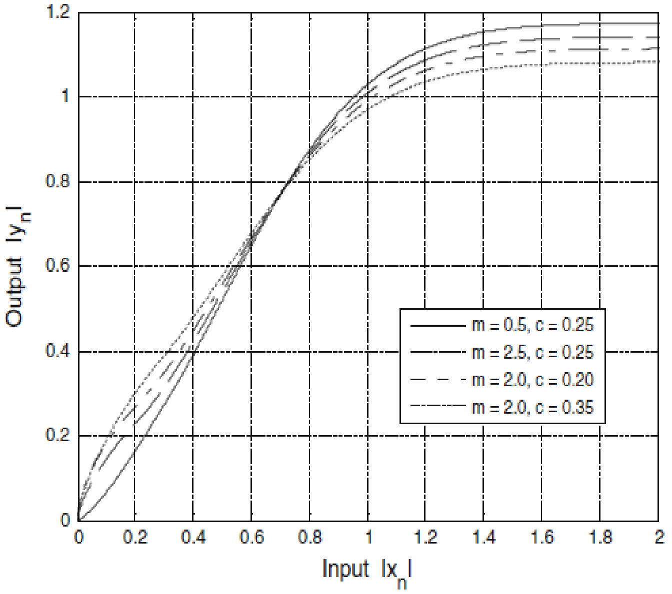

Fig. 2. Transfer curves with various transform parameters

According to the transfer curves in Fig. 2, the proposed scheme compresses large signals while partially enlarging small ones. Also a constant average power level is maintained in the transform which is an advantage. Hence, the PAPR can be reduced more efficiently and the system can achieve immunity from the channel noise to small signals.

Theoretical result of the final PAPR of the transformed signal y in the proposed scheme is depicted in Fig. 3. Results show that along with the increase in m or c the proposed scheme can obtain more PAPR reduction. In the proposed scheme the PAPR can effectively be confined in the interval [2.18 dB, 2.95 dB] by adjusting the transform parameters.

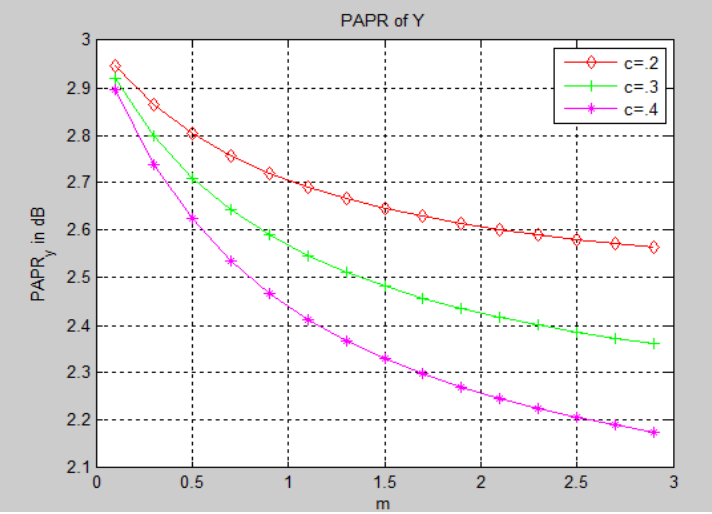

Fig. 3. Theoretical results of PAPR

5 SIMULATION RESULTS

Evaluation of the proposed scheme is performed through numerical simulations and expressed in terms of the reduction in PAPR and BER performance. An un-coded OFDM system with N = 1024 and QPSK or 16QAM modulation is considered for evaluation. The over-sampling factor is selected as l = 4. To investigate the performance degradation, the transformed signals are passed through an Additive Gaussian White Noise (AWGN) channel and through a Rician Fading Channel. For the puepose of comparison, the simulation results obtained for the classic schemes including the μ-law[7], EC[9], PC[10], and TC[11],[12] were considered.

5.1 PAPR Reduction

The CCDFs of the original and transformed OFDM signals with 16QAM using the proposed NCT scheme is illustrated in Fig. 4.

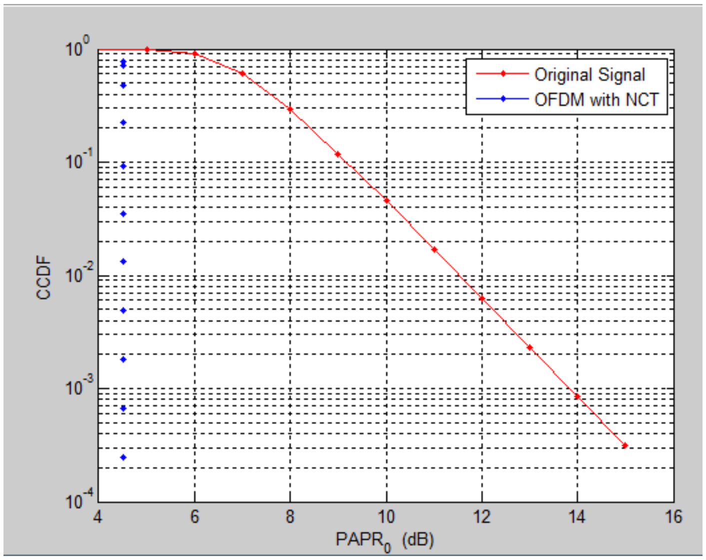

Fig. 4. The CCDF statistics of the original and transformed OFDM signals

Simulation results show that by using the proposed scheme, for $CCDF = 10^{-3}$, the PAPR can be reduced to 9.2dB with *m* = *0.5*. From the results it is clear that in the proposed scheme flexibile PAPR levels can be achieved by adjusting the transform parameters. Which is an additional advantage compared to the reported methods.

5.2 BER

Simulation results for BER of different schemes using 16QAM over AWGN channel and the Rician fading channel are described. Fig. 5.a depicts the BER versus E_b/N_0 curves over an AWGN channel and Fig. 5.b depicts the BER versus E_b/N_0 curves over the Rician fading channel.

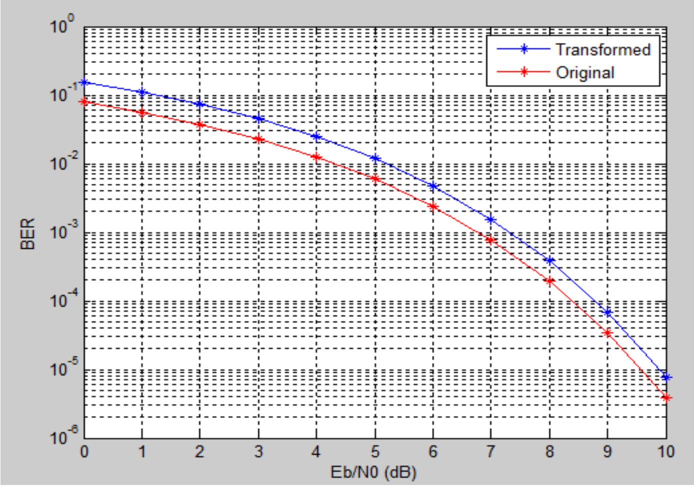

Fig. 5.a. BER for 16QAM OFDM system over AWGN channel

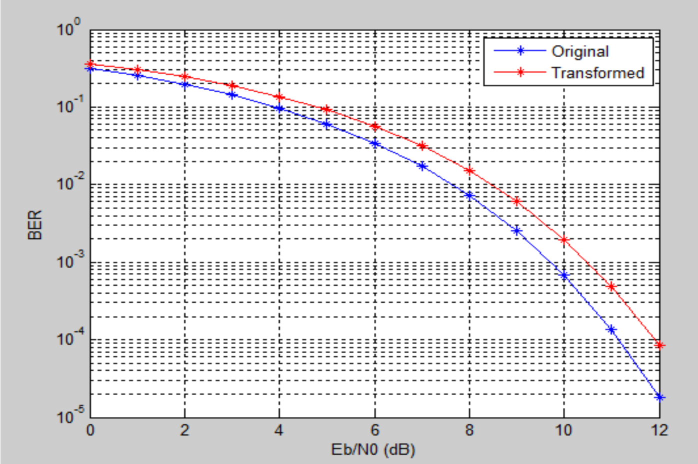

Fig. 5.b. BER for 16QAM OFDM system over the Rician fading channel

Simulation curves shows that, all the curves corresponding to the transformed signals are located to the right of the 'original signal' curve, which means that NCT operation causes certain signal attenuation. However from Fig. 5.a and Fig. 5.b, it can be seen that the proposed scheme can achieve better BER performance compared to the reported schemes at the desired PAPR level.

Table 1 summarizes the simulation results of the PAPR reduction and BER performance with different companding schemes reported earlier. From the results, when compared with the reporte works, the proposed scheme with *m* = *0.5* provides better BER performance and lower out-of-band radiation to reach a given PAPR level. Unlike the EC [9] and PC[10] schemes whose performances are almost fixed; the proposed scheme is flexible in the companding form. Hence the proposed scheme can obtain an efficient tradeoff between PAPR reduction and BER.

TABLE 1

COMPARISONS BETWEEN PAPR AND BER WITH DIFFERENT NCT SCHEMES USING 16QAM

Companding schemes	E_b/N_0 (dB), for 16QAM		PAPR(dB) at CCDF = 10^{-3}
	AWGN (BER = 10^{-5})	RICIAN (BER = 10^{-4})	
Original signal	8.3	11	13.9
µ-law in [7]	10.42	13.28	5.58
EC in [9]	11.34	13.74	4.88
PC in [10]	9.28	13.43	4.73
TC in [11]	10.15	12.74	4.69
TC in [12]	9.40	12.64	5.24
Proposed scheme with m = 0.5	8.8	11.9	4.5

In the proposed scheme in Fig. 5.a, for an OFDM system with 16QAM over an AWGN channel with *m* = *0.5*, BER of 10^{-5} is achieved at E_b/N_0 of 8.8dB. This is 0.6dB to 2.54dB greater than others. In Fig. 5.b, for an OFDM system with 16QAM over a Rician fading channel with *m* = *0.5,* BER of 10^{-4} the achieved at E_b/N_0 of 11.9dB. This is 0.74dB to 1.38dB greater than others. Thus the proposed scheme results in less impact of companding distortion on the BER performance than others.

6 CONCLUSION

A novel NCT scheme for reducing the PAPR of OFDM signals has been presented and evaluated. In the proposed scheme the original Gaussian-distributed OFDM signal is transformed into a specific statistics form defined by a piecewise probability density function with an inflexion point. According to theoretical analysis, the impact of companding distortion on the BER performance may be considerably decreased to reach a given PAPR level. A favorable tradeoff between the PAPR reduction and BER performance is also provided by this scheme. The analytical expressions in terms of the PAPR, ultimate CCDF and signal attenuation coefficient are also derived. Simulation results shows that, with the proposed scheme, the transformed signals have three main advantages: less signal distortion, lower out-of-band radiation and more flexible companding form, when compared to the classic NCT techniques.

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